PLTW Math Frameworks

PLTW Course: Civil Engineering and Architecture (CEA)

Math Strand being addressed: Geometry

Math Standard being addressed: 9.3.4

Overview:

Math Standard and Benchmarks: 9.3.4.2, 9.3.4.3, 9.43.4.7

Standard 9.3.4: Solve real-world and mathematical geometric problems using algebraic methods.

Benchmark 9.3.4.2: Apply the trigonometric ratios sine, cosine and tangent to solve problems, such as determining lengths and areas in right triangles and in figures that can be decomposed into right triangles. Know how to use calculators, tables or other technology to evaluate trigonometric ratios.

Benchmark 9.3.4.3: Use calculators, tables or other technologies in connection with the trigonometric ratios to find angle measures in right triangles in various contexts.

Benchmark 9.3.4.7: Use algebra to solve geometric problems unrelated to coordinate geometry, such as solving for an unknown length in a figure involving similar triangles, or using the Pythagorean Theorem to obtain a quadratic equation for a length in a geometric figure.
(Minnesota Department of Education, 2010)

Correlation to Common Core Math Standards:

N-VM: Represent and model with vector quantities.
1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $\|\boldsymbol{v}\|$, v).
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. Solve problems involving velocity and other quantities that can be represented by vectors.

N-VM: Perform operations on vectors.
4. Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. Multiply a vector by a scalar.
a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|c\boldsymbol{v}\| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

G-SRT: Define trigonometric ratios and solve problems involving right triangles
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G-MG: Apply geometric concepts in modeling situations
1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
(Common Core State Standards Initiative, 2010)

References:

Common Core State Standards Initiative. (2010). *The Standards, Mathematics.* Retrieved July 01, 2011, from Common Core State Standards Initiative: http://www.corestandards.org/the-standards/mathematicMinnesota Department of Education. (2010, May 24). *Academic Excellence: Academic Standards: Science.* Retrieved July 1, 2011, from Minnesota Department of Education: http://education.state.mn.us/mdeprod/groups/Standards/documents/Publication/013906.pdf

Essential Understandings/Big Ideas:

* In Civil Engineering and Architecture, the type of shape used is critical for the strength and the aesthetic view of the structure.
- Triangles are the most common shape used by engineers. People will find them in roof designs and bridge designs.
- It is a necessity for engineers and architects to solve problems relating to triangles and other common shapes.

In CEA, students will use trigonometry to find slopes of roofs and lengths of rafters. Along with the trigonometry, students will use algebra to find surface areas of roofs,

floors, and walls. The concept of vector forces will be applied on beams, columnss and footings. The concepts of trusses can be expanded in Unit 2 Roof Systems and in Unit 3 Structures. All these tasks will be done by use of calculator, but also with the aid of modeling software, such as MDSolids.

What should students know and be able to do [at a mastery level] related to these benchmarks?

Students will be able to:

- Know the components of a right triangle.
- Solve problems using trigonometry ratios of sine, cosine and tangent.
- Apply formulas, such as Pythagorean Theorem, to find surface areas, volume, and cost.
- Understand the x and y components of right triangles.
- Apply the concept of vectors to find the forces acting upon a beam, column, or footing.
- Use special modeling software to aid in solving right triangle problems.

Misconceptions:

Student Misconceptions

- Students believe trigonometry is only performed in math class.
- Students do not need to apply mathematical problem solving in technology and engineering classes.
- Students believe a force is something that you only apply on a wrench or a vice.

Teacher Resources:

Teacher Notes

Students will explore and gain an understanding of:

- Where does it fit?
- Right triangles, its trigonometric ratios, properties and related formulas, are used throughout Architecture and Engineering. In this course, teachers will find a heavy use in Unit Lesson 2.1, 2.2, and 3.2.

How to best introduce the concept?

1. Define key terms when appropriate.
2. Use and label a model when defining these terms.
3. Identify the right triangle in relationship of your model (i.e. roof).
4. Review the trigonometry ratios of sine, cosine, and tangent, show the relation to the triangle.
5. Show the relationship between slope and tangent.
6. Create a handout of the formulas to help aid in understanding of them.
7. Use the modeling software, MDSolids, to model and calculate truss and beam properties.
8. Follow lessons 2.1, 2.2, and 3.2, remember you can enhance the activity.

What students struggle the most?
Students tend to have a tough time understanding and applying the mathematics along with the new terminology of the course. For some students, it is the first time they may have been introduced to this content. These students tend to be more visual learners. To aid in their learning, use demonstrations and physical or computer generated models. Be sure to describe the connection between the model and the concept. It is best to create more practice problems that will enforce the concepts, functions, and formulas. Use formative assessments to see where the students still struggle and re-teaching maybe required.

New Vocabulary

Pitch: It is the slope of a roof. Mathematically, pitch equals the total rise divided by the total span. But in industry, the term pitch is used to describe the slope of the roof as in 4/12 pitch, and the convention for construction is that the denominator will always be 12. This implies a roof will rise four inches to every 12 inches of run. Slope equals rise over run. It is simpler for contractors to refer the slope of a roof as a 4/12 pitch. Slope refers to the angle of a line. So, we can make the following connection: slope of a line = rise/run = opposite/adjacent = tangent of the angle between roof line and the run. This formula can now be used to measure the angles out on rafters.

Load: Forces or other actions that result from the weight of building materials, occupants and their possessions, and environmental effects. These are the vector forces that are applied to structures. They are used to calculate the strength of the structure.

Vignette:

The class begins with a presentation on roof types from the instructor. Different roof types were discussed along with key terminology. Pitch, slope, run, and rise were some of the words. Using standard gable roof, the instructor began to model the right triangle that contains the slope, rise, and run. The instructor identifies all the terms related to a right triangle including the trigonometry ratios.

The students are given a practice worksheet with some sample problems. Two problems were demonstrated by the instructor. The first one made the connection with rise over run and tangent. This gave the angle the roof makes with the run. The second problem showed how to calculate the length of the rafter. The instructor allows them to work on the problems for the remainder of the class period.

The next class period the instructor describes different types of roof trusses. A truss specification sheet is given to the students for references. Along with the specification sheet and building materials, the instructor assigns Project 2.1.2. Project 2.1.2 had the students build a model of a roof with a given pitch and run. The students begin to calculate the size of the materials to make their trusses using the trigonometry functions discussed the day before. After they calculated what they needed, the students build their model roofs to size. Once their model is complete, they need to estimate the amount of building materials needed and the cost to build their roof in real life.

Additional Instructional Resources

Any Advanced Algebra, Pre-Calculus, or Trigonometry textbook would be a great resource to find more information on right triangle properties, ratios, and formulas. Your local mathematics department should have those.

Civil Engineering & Architecture Matteson, Kennedy, Baur, ©2012ISBN-13: 978-1-4354-4164-4 Delmar, Cengage Learning PO Box 6904, Florence, Kentucky 41022t: 800-354-9706f: 800-487-8488.

Civil Engineering & Architecture Matteson, Kennedy, Baur, ©2012WorkbookISBN-13: 978-1-4354-4165-1Instructor ResourcesISBN-13: 978-1-4354-4166-8Architectural Graphic Standards, 11th Ed. The American Institute of ArchitectsISBN-13:978-0-471-70091-3 Delmar, Cengage Learning PO Box 6904, Florence, Kentucky 41022t: 800-354-9706f: 800-487-8488

Revit – 2011 tutorial books – go to www.leadinghighschools.com to receive a complimentary examination copy

Architectural Drafting and Design – ISBN-13 978-1-4018-6715-7 Alan Jefferis and David Madsen, Thomson, Delmar Learning Jeanne.Blair@cengage.com is the district rep and can provide information for you about the textbooks and can give you examination copies. Here is the website to get the examination copy http://cengagesites.com/academic/?site=3243

Revit Architecture – ISBN-13: 978-1-4390-5715-5 Author Paul Aubin (also from cengage.com)

Francis D. K. Ching several books on architecture John Wiley & Sons, Inc. publisher
Architecture: Form, Space and Order, ISBN 978-0-471-75216-5
Architectural Graphics ISBN 0471209066
Building Construction Illustrated ISBN 0-471-35898-3

Websites:
http://regentsprep.org/REgents/math/ALGEBRA/AT2/Ltrig.htm
http://www.youtube.com/watch?v=dLOdHzttQ90
http://www.youtube.com/watch?v=NTuQWko0PfY
http://www.clarku.edu/~djoyce/trig/right.html
http://library.vcc.ca/learningcentre/pdf/worksheets/Drafting-RoofSlopeandBevelRatio.pdf
http://roofgenius.com/roofpitch.htm
http://www.roofingcalculator.org/roof-pitch.php
http://www.physicsclassroom.com/class/vectors/u3l3c.cfm
http://www.sciencebuddies.org/science-fair-projects/project_ideas/CE_p006.shtml

Below is a list of Civil Engineering and Architecture activities and/or projects that incorporates the standards.

Project 2.1.2 Roof Systems – Students will build a model of a roof system. A list of materials and tools is given in the project documentation. Additionally, students will need a ruler and a calculator to calculate angles and sides of triangles. Students are required to

show all work in the plan of their roof. The instructor can very easily expand this project to include surface area of roof, trusses unique to each group, and a cost estimation of such roof design. It is our recommendation to expand and increase learning in designing roofs. Refer to the instructor notes given within the curriculum for further points of emphasis.

Activity 2.2.2 Shed Cost Estimation – In this activity, students must estimate the cost of a utility shed. This includes the estimate of a gable style roof. Currently, the activity allows the student to use Revit to calculate the surface area of the roof. This concept can be expanded to have students calculate by hand first. Then use the software to verify their answer is correct. Not all contractors have a computer with special software on site; they still need to estimate by hand.

Lesson 3.2 Structures – This lesson contains many activities. Their main focus is Statics, which is the physical study of keeping objects motionless. Since most of the loads are horizontal and vertical, students may not use trigonometry. But, the size, shape and length of span of structural members do have an impact of how the loads are carried. Students will use cross-sectional areas, volumes, material properties, and structural formulas to determine the strength of the structure. Students will design beams and spread footings. If a student's design has trusses, then that student will need to apply trigonometry functions to determine its load bearing capabilities. All skills acquired in Lesson 3.2 are applied for the Keystone Library Renovation Project at the end of Unit 3.

Assessment:

Assessment Methods

Methods to incorporate explanation, application and perspectives:

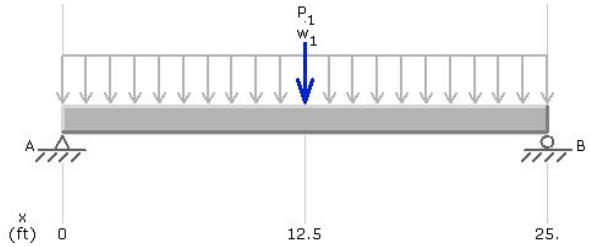
1. At what angle does the roof make to the horizon if the pitch is 4/12 and the span of the roof is 24 feet? Answer: $\tan^{-1}(4/12) = 18.43°$

2. If a truss spans 30 feet and the rises 6 feet in the middle, what is the length of the rafter the carpenter needs to cut, ignoring overhang? Answer: First, find the run: 30'/2 = 15', second, use Pythagorean Theorem: $\sqrt{(15^2 + 6^2)} = 16.16'$

3. What angle does the plumb cut need to be on a rafter with a 6/12 pitch? Answer: $\tan^{-1}\left(\frac{6}{12}\right) = 26.6$

4. Which of the following loads is **not** considered a Dead Load?
   A. roof structure ***C.* occupants**
   B. floor tile ***D.*** walls

Use the load diagram and description below to answer the questions.



Note: w1= 100 lb/ft and P1= 500 lb

5. In the beam loading diagram above, what is the reaction at Support A?
   A. 250 lb *C.* 1,250 lb

   B. 500 lb ***D.* 1,500 lb**

6. A 60,000-pound load from a column is placed on a 4 foot by 4 foot spread footing. What is the soil bearing pressure due to the load?
   A. 16 sf C. 960 ksf

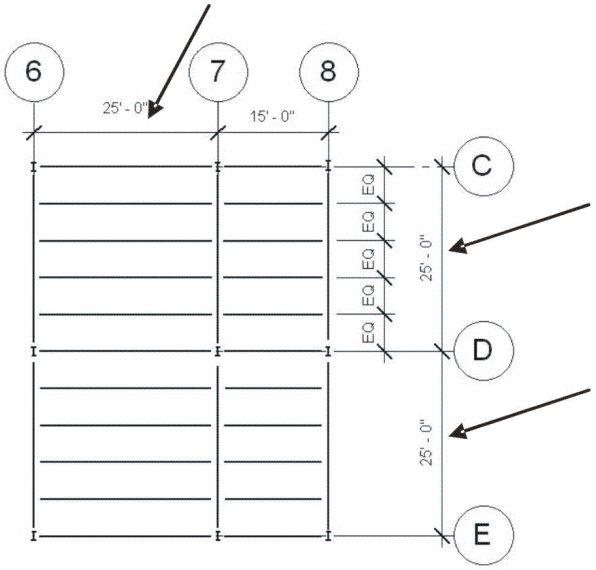
   B. 600 lb **D. 3,750 psf**

7. Below is the preliminary structural steel layout of building. The owners and the architect have agreed to revise the column spacing, in both the bay between Column 6 and 7 and the bays between Columns C to D and D to E (bays noted for clarity).

   The span length will change from 25 feet to 30 feet. The number of beams in the bays will not change.

   List 5 structural items that will be affected by this design change**.**

   (5 points total: 1 point each)



Possible answers:
Tributary width of the beams will change thus increasing the load on that beam
Span length of beam increases
Longer Beams, same load, therefore End Reactions will increase

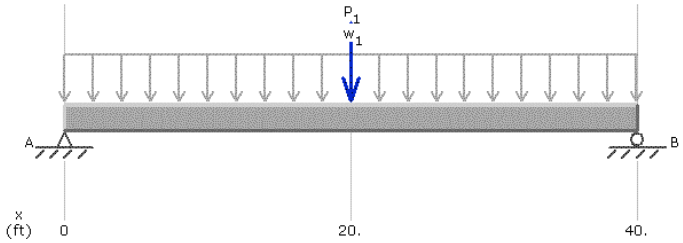
Moment in beams will increase, therefore increase beam sizes
Footing size will increase, as the tributary floor area has increased for the column/footing

8. Beam Design

a) Calculate the End Beam Reactions and the Maximum Moment for the following beam. All work must be shown.

a. Calculate the required Section Modulus for the beam.

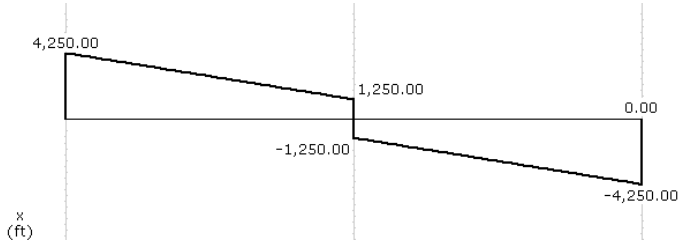
Span = 40 feet
W_1= 150 lb/ft
P_1= 2,500 lbs. at 20 feet from left support
Use an Allowable Bending Stress of $F_b = 24{,}000$ lb/in^2



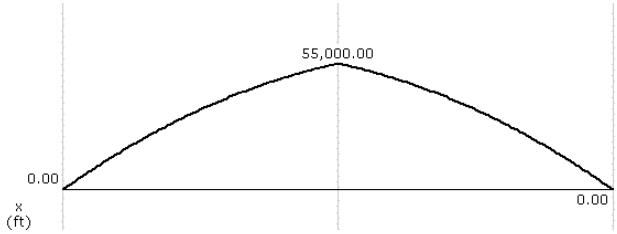
(5 points total: 2 points for R_A and R_B; 2 points for Moment and 1 point for Section Modulus)

Accuracy (0)

a. Reactions : Note that the beam is symmetrically loaded:
$R_A = R_B = wL/2 + P/2$
$R_A = R_B = [(150 \text{ lb/ft})(40\text{ft})]/2 + [2500\text{lb}/2]$
R_A= 4,250 pounds = R_B = 4,250 pounds



Moment: Maximum Moment occurs at midspan
$M = wL^2/8 + PL/4$
$M= [(150 \text{ lb/ft})(40\text{ft})^2]/8 + [(2{,}500 \text{ lb})(40\text{ft})/4]$
M= 30,000 lb -ft + 25,000 lb-ft
M= 55,000 lb-ft



b. Section Modulus

$S_x = M/F_b$
$S_x = [(55{,}000 \text{ lb-ft})(12 \text{ in/ft})] / 24{,}000 \text{ lb/in}^2$
$S_x = 28 \text{ in}^3$

Footing Design (5 points total)

Accuracy (0.00)

Size a Square Footing given the following information:

- Design Loads
  - Dead Load =35 lb/ft^2
  - Live Load =50 lb/ft^2
- Tributary Floor Area Carried by Footing = 40 ft x 40ft
- Allowable Soil Bearing Capacity 2,500 lb/ft^2
- Assume thickness of footing is equal to 2'-0"
- Weight of concrete 150 lb/ft^3

SHOW ALL WORK

(DL + LL) Tributary Area= (35 lb/ft^2 +50 lb/ft^2) (1,600ft^2) = 136,000 lb

Pressure = Load /Area q=P/A

Weight of Footing = (2ft) x 150 lb/ft^3 = 300 lb/ft^2

q_{net} = Soil capacity Available= 2,500 lb/ft^2 – 300 lb/ft^2 = 2,200 lb/ft^2

Footing Area Required = P/ q_{net}

A=136,000 lb /2,200 lb/ft^2

A=61.82 ft^2

Use 8 ft X 8 ft square footing

Differentiation:

Gifted and Talented

Gifted students can expand this concept by designing and testing their own roof truss system. They need to calculate the size and strength of all truss members as would all structural engineers do. The students should build and test a model to see how accurate their theoretical designs are. This testing can be done with the stress analyzer. MDSolids can be used to help proof their designs.

Special Education

All students typically will work in groups of two or three. This should help as the assignment load is divided out amongst all the members. Each special education student should be paired with another student that has or shows ability to master the mathematical concepts. Most work is hands on with the computer and other tools. This typically aids in their understanding. Additional practice in the mathematic concepts should be offered for more clarity and understanding. Coordination with the individual's case manager would be extremely helpful for any modifications to assignments and assessments. Formative assessments should be used for planning and the needs of the students.

English Language Learners

Mathematics is a universal language. The difficulty is the English vocabulary that accompanies it. Post lessons, lectures, activities, and vocabulary with definitions on a class website or an online instructional tool such as Moodle. This will allow ELL students the opportunity to review the vocabulary at their convenience. Translations of documents can be formed by the ELL instructor within your school district. There are cases of students who lack a mathematical foundation. Those students need to be assessed so the proper instruction can be given. Each student is different. Your ELL instructor is your best support.

Parents and Administration:

Administrative/Peer Classroom Observation

Students Are: (descriptive list)	**Teachers Are:** (descriptive list)
Using Pythagorean Theorem to solve lengths of sides on right triangle.	Demonstrating how to model and calculate parts of a right triangle
Using trigonometry ratios to find angles of slopes or joints to be made on rafters.	Demonstrate how to use trigonometry ratios.
Finding surface area of roofs, walls, and floors for cost estimation	Demonstrate how to calculate loads on structures.
Apply formulas to calculate loads, moments, and vector forces on trusses, beams, columns, or footings.	Moving around the classroom, helping groups with the activities and projects.
Assembling models of roofs for testing.	Using correct math vocabulary when appropriate.

Professional Learning Communities:

Reflection – PLTW teachers should become familure with trigonmetric ratios. They should also have discussions with Math and Science instructors on the use of vectors.

Materials – PLTW teachers should share the above additional instructional resources with other instructors during their discussion.

Parent Resources:

Refer parents to the suggested lists and searches found in Additional Instructional Resources.

It would be very helpful to create a website in which the instructor can communicate with parents on assignments, topics, and additional support.

References:

Common Core State Standards Initiative. (2010). *The Standards, Mathematics.* Retrieved July 01, 2011, from Common Core State Standards Initiative: http://www.corestandards.org/the-standards/mathematics

Minnesota Department of Education. (2010, May 24). *Academic Excellence: Academic Standards: Science.* Retrieved July 1, 2011, from Minnesota Department of Education: http://education.state.mn.us/mdeprod/groups/Standards/documents/Publication/013906.pdf