

# PLTW Math Frameworks

**PLTW Course:** Principles of Engineering

**Math Strand being addressed:** Algebra

**Math Standard being addressed** 9.2.2

## Overview:

**Math Standard and Benchmarks:** 9.2.2.1

**Standard 9.2.2:** Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.

**Benchmark 9.2.2.1:** Represent and solve problems in various contexts using linear and quadratic functions.

**Correlation to Common Core Math Standards:**

MN 9.2.2

F,IF,4,5,6

## Essential Understandings/Big Ideas:

- Material Testing is a critical process that determines whether a product is reliable, safe, and predictable in function.
- Material testing is basically divided into two major categories: destructive testing and nondestructive testing.
- Destructive testing is defined as a process where a material is subjected to a load in a manner that will ultimately cause the material to fail.
- Machines have been developed specifically to conduct destructive testing.
- Some of these machines exert force on the sample and record information such as resulting deformation, the amount of stress that builds up inside the sample, elastic behavior, strength, etc.
- When non-destructive testing is performed on a material, the part is not permanently affected by the test. The part is usually still serviceable.
- The purpose of non-destructive testing is to determine whether the material contains imperfections.

Over many years, tests have been developed for measuring the common properties of engineering materials including: acoustical, electrical, magnetic, physical, optical, and thermal properties.

## **What should students know and be able to do [at a mastery level] related to these benchmarks?**

### **Students will be able to:**

- Utilize a five-step technique to solve word problems.
- Obtain measurements of material samples.
- Tensile test a material test sample.
- Identify and calculate test sample material properties using a stress strain curve.

## **Misconceptions:**

### **Student Misconceptions**

The packet used for Activity 2.3.2 is used with the results from the SSA graph produced from the destructive test of the “dogbones”.

- Students will confuse the locations on the graph to use for data.
- It is suggested that instructors “walk” through the calculations of a sample dogbone of some other material.
- Provide the graph and then work together as a class to determine how calculations are made from the graph and which data to use.

## **Teacher Resources:**

### **Teacher Notes**

Students will explore and gain an understanding of:

- Forces acting on a body in static equilibrium.
- Calculating internal and external forces of a system.
- Basic categories and properties of materials.
- Material testing.
- Design problems related to materials and structures

The instructor should practice the material testing procedure.

It is important that the materials are tested in the same way to give students similar results to one another as well as to established standards.

The instructor should provide a graph of the data in digital form so students can import into their own document and manipulate.

The graph is used for all calculations and thus important to be of an adequate size to be read and understood.

A student’s attempt to get any answer often results in overlooking what would seem obvious to the instructor.

The instructor should perform a practice destructive test using some other material than brass or aluminum. Students should perform calculations with the instructor to understand the process and use of the graphic data to make calculations.

The modulus of Elasticity formula is given as:

$$E = \frac{(P1 - P2)L0}{(\text{total deformation1} - \text{total deformation2})A}$$

When solving for modulus of Elasticity mark points P1 and P2 directly on the graph. P1 should be higher on the graph and P2 lower for ease in calculations. (less working with negative numbers).

Create a straight line to indicate the slope of the line directly on the graph connecting points P1 and P2. This will enable students to see what the slope actually is.

Total Deformation 1 is directly below vertically from P1 on the chart and will be larger than Total Deformation2, directly below vertically from P2.

L0 is the original length of the narrow region and A is the original area of the narrow region. Be sure students match units so they cancel correctly.

After students calculate the Modulus of Elasticity, compare the results to known values used by engineers and designers. Student values should be similar. Remember there are a variety of Aluminum alloys. A common alloy used in general purposes is 6061.

### Additional Instructional Resources

Principles of Engineering, 1st Edition Brett Handley - Wheatfield-Chili Middle School/High School, New York, Craig Coon, David M. Marshal, SBN-10: 1435428366 ISBN13: 9781435428362.

The text, Principles of Engineering, by Handley, Coon, Marshall may be purchased for use by students both in and out of class as a resource.

### New Vocabulary

Axial Stress	A force with its resultant passing through the centroid of a particular section and being perpendicular to the plane of the section. A force in a direction parallel to the long axis of the structure.
Breaking Stress	The stress required to fracture a material whether by compression, tension, or shear.
Compression	When a material is reduced in volume by the application of

	pressure; the reciprocal of the bulk modulus.
Deformation	Any alteration of shape or dimensions of a body caused by stresses, thermal expansion or contraction, chemical or metallurgical transformations, or shrinkage and expansions due to moisture change.
Destructive Testing	Test methods used to examine an object, material, or system causing permanent damage to its usefulness.
Elastic Limit	Maximum stress that a material will withstand without permanent deformation.
Elongation	The fractional increase in a material's length due to stress in tension or thermal expansion.
Failure Point	Condition caused by collapse, break, or bending, so that a structure or structural element can no longer fulfill its purpose.
Fatigue	The loss of the load-bearing ability of a material under repeated load application, as opposed to a single load.
Modulus of Elasticity	The ratio of the increment of some specified form of stress to the increment of some specified form of strain, such as Young's modulus, the bulk modulus, or the shear modulus. Also known as coefficient of elasticity, elasticity modulus, elastic modulus.
Nondestructive Testing	Test methods used to examine an object, material, or system without impairing its future usefulness.
Proportional Limit	Point at which the deformation is no longer directly proportional to the applied force. Hooke's Law no longer applies.
Shear Stress	A measure of how easily a material can be twisted.
Standard Deviation	A statistical measurement of variability.
Strain	Change in the length of an object in some direction per unit.
Stress	The force acting across a unit area in a solid material resisting the separation, compacting, or sliding that tends to be induced by external forces.
Stress-Strain Curve	Graphical representation of a material's mechanical properties.

Tension	The condition of a string, wire, or rod that is stretched between two points.
Toughness	Mechanical property of a material that indicates the ability of the material to handle overloading before it fractures.
Ultimate Stress	Sometimes referred to as tensile strength; determined by measuring the maximum load a material specimen can carry when in the shape of a rectangular bar or cylindrical can.
Variance	The average of the squared differences from the mean.

## Vignette:

Background: Students have been testing their “dogbones” using the SSA machine. Results from each test provide the students with a stress/strain graph. The teacher instructs the students to examine the graph for the aluminum dog bone.

The teacher explains, “You will notice on your graph that there are two distinct regions. There is a diagonal line that goes up and to the right. Near the top of the straight line the line begins to curve to a peak and then recede. This curved area forms an upside down bowl. This is a parabola. We are going to use mathematical equations to determine several attributes about these two regions of the graph.”

The teacher asks the class, “Does anyone know what type of function or equation the straight line represents?”

A student may respond, “Is it linear?”

“Correct. An example of a linear equation is  $f(x) = 3x - 4$  where the function  $f$  has a slope of 3 and a  $y$ -intercept of -4.”

The instructor points out, “ In the case of our line we don’t cross the  $x$  or  $y$  axis and can’t write a linear equation to represent it.” However, we can also calculate the slope of a line using the formula:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

“You are going to draw a straight line that is a best fit to the graph line.

Mark two points on the best fit line a good distant apart.

It is easiest calculating if you choose an  $x,y$  location for each point that lines up with number from the graph.”

The teacher demonstrates the procedure using a interactive whiteboard, document camera, or overhead projector.

The teacher then moves around the room making sure students have correctly drawn a line of best fit, and place to points on the line that provide reliable numbers on the x and y axis.

The teacher continues, “Now label the point that is further up and to the right on the line as 1 and the lower point as 2.”

The teacher asks the class, “What can we do with these two points?” A student responds, “I can take the x and y coordinates at each point and put them into the formula for calculating slope.”

“That’s exactly what we are going to do,” replies the teacher. Students work to determine the slope of their best fit line.

“Now I would like you to look at another equation. It looks like this.”

The teacher displays the formula,  $E = (P_1 - P_2)L_0 / (\delta_1 - \delta_2)A$ , to the class.

“Looking at the graph and the formula, what can anyone tell me about points 1 and 2 and the new formula?”

The students consider for a while but are hesitant in answering.

The teacher answers their own question. “Just as we solved the slope of the line with points 1 and 2. We can solve this equation with points 1 and 2 as well. What is the difference in the equations?”

A student volunteers, “the second formula is essentially the same as the slope formula, except that the Y or P locations are multiplied by  $L_0$ . The X or  $\delta$  locations are multiplied by A.”

“Exactly,” comments the instructor. “The second formula is for solving an items Modulus of Elasticity. The result of the original length of the test location multiplied by the difference between points 1 and 2 vertically, divided by the original cross sectional area of the test location multiplied by the difference between points 1 and 2 horizontally.”

“So the Modulus of Elasticity is basically the slope of a line on a graph, with relation to the items size (length and area) before testing?” states a student.

“That’s a good way to put it,” states the instructor.

“Now to show that it is indeed a linear function we could draw an x and y axis on the graph that intersects our best fit line. Using the calculated slope (m) from earlier we can place numbers into the formula  $f(x) = mx + c$ , where m is the slope, and c is the y-intercept location.”

“Tomorrow we will look at Quadratic functions and see how they can be useful in analyzing the second area of our graph. For now, let’s take out the brass stress/strain graph and determine the slope and the Modulus of Elasticity,” directs the instructor.

Students use the example provided by their aluminum “dogbone” stress/strain graph to complete similar calculations on their brass “dogbones” stress/strain graph.

## **Assessment:**

### **Assessment Methods**

#### **Explanation**

Students will explain the importance of material testing as a verification process.

#### **Application**

Students will tensile test a material sample and identify and calculate material properties.

#### **Interpretation**

Students will write journal entries reflecting on their learning and experiences.

Example writing prompt:

What is something you learned today about material testing, manufacturing processes, or engineering problem solving that you did not understand or know before?

#### **Self-Knowledge**

Students will reflect on their work by recording their thoughts and ideas in journals. Students may use self-assessments as a basis for improvement.

Ideas and questions students may pose and respond to in their journals are:

- Today the hardest concept for me to understand was . . .
- When I work in a group, I find that . . .
- When I work by myself, I find that . . .
- What did I accomplish today?
- Now that I have completed this task, what is next?

## **Differentiation:**

### **Gifted and Talented**

Students who understand the concepts can be supplied with a stress-strain curve graph from which they can calculate axial stress, shear stress, total deformation, normal Strain, modulus of elasticity and modulus of toughness. Students would use their results to try and determine what the material is composed of.

### **Special Education**

For students on an IEP; all PowerPoints, reading materials and activity sheets can be made available on a school sponsored, secure web site. Teachers using interactive whiteboards and other enhancements make lessons available on a website. Students could review work outside of class time to reinforce their understanding.

## English Language Learners

Vocabulary should be an integral part of the unit. The instructor can use several different strategies to introduce terminology such as a word of the day with weekly and unit quizzes to reinforce learning.

## Parents and Administration:

### Administrative/Peer Classroom Observation

Students Are:	Teachers Are:
Taking notes and completing practice tensile tests calculations.	Demonstrating the safe and effective use of tensile testing equipment.
Using a lathe to make “dogbones.”	Demonstrating and supervising the use of lathes.
Testing their “dogbones” with the structural stress analyzer.	Providing students the digital data from the SSA to use in performing calculations.
Analyzing data and performing calculations from the graphical information.	Assisting students in performing data interpretation and calculations.
Comparing data and calculations with “knowns.”	Analyzing student work.

## Professional Learning Communities:

### Reflection:

Critical Questions regarding the teaching and learning of these benchmarks:

1. Why is it critical for engineers to document all calculation steps when solving problems?
2. How is material testing data useful?
3. Stress strain curve data points are useful in determining what specific material properties?

### Materials:

Suggested articles and books for PLC book study are:

Principles of Engineering, 1st Edition, Brett Handley - Wheatfield-Chili Middle School/High School, New York, Craig Coon, David M. Marshal, SBN-10: 1435428366  
ISBN-13: 9781435428362.

## Parent Resources:

Tensile Testing. Wikipedia, [http://en.wikipedia.org/wiki/Tensile\\_testing](http://en.wikipedia.org/wiki/Tensile_testing).

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